Chapter 5. Factorisation

Ex 5.1

Answer 1A.

$4x^2y^3 - 6x^3y^2 - 12xy^2$

Here, the common factor is $2xy^2$.

Dividing throughout by $2xy^2$, we get

$$\frac{4x^2y^3}{2xy^2} - \frac{6x^3y^2}{2xy^2} - \frac{12xy^2}{2xy^2}$$

$= 2xy - 3x^2 - 6$

$\therefore 4x^2y^3 - 6x^3y^2 - 12xy^2 = 2xy^2(2xy - 3x^2 - 6)$

Answer 1B.

$5a(x^2 - y^2) + 35b(x^2 - y^2)$

Here, the common factor is $5(x^2 - y^2)$.

Dividing throughout by $5(x^2 - y^2)$, we get

$$\frac{5a(x^2 - y^2)}{5(x^2 - y^2)} + \frac{35b(x^2 - y^2)}{5(x^2 - y^2)}$$

$= a + 7b$

$\therefore 5a(x^2 - y^2) + 35b(x^2 - y^2) = 5(x^2 - y^2)(a + 7b)$

Answer 1C.

$2x^5y + 8x^3y^2 - 12x^2y^3$

Here, the common factor is $2x^2y$.

Dividing throughout by $2x^2y$, we get

$$\frac{2x^5y}{2x^2y} + \frac{8x^3y^2}{2x^2y} - \frac{12x^2y^3}{2x^2y}$$

$= x^3 + 4xy - 6y^2$

$\therefore 2x^5y + 8x^3y^2 - 12x^2y^3 = 2x^2y(x^3 + 4xy - 6y^2)$

Answer 1D.

$12a^3 + 15a^2b - 21ab^2$

Here, the common factor is 3a.

Dividing throughout by 3a, we get

$\frac{12a^3}{3a} + \frac{15a^2b}{3a} - \frac{21ab^2}{3a}$

$= 4a^2 + 5ab - 7b^2$

$\therefore 12a^3 + 15a^2b - 21ab^2 = 3a(4a^2 + 5ab - 7b^2)$

Answer 1E.

$24m^4n^6 + 56m^6n^4 - 72m^2n^2$

Here, the common factor is $8m^2n^2$.

Dividing throughout by 3a, we get

$\frac{24m^4n^6}{8m^2n^2} + \frac{56m^6n^4}{8m^2n^2} - \frac{72m^2n^2}{8m^2n^2}$

$= 3m^2n^4 + 7m^4n^2 - 9$

$\therefore 24m^4n^6 + 56m^6n^4 - 72m^2n^2 = 8m^2n^2(3m^2n^4 + 7m^4n^2 - 9)$

Answer 1F.

$(a-b)^2 - 2(a-b)$

Here, the common factor is $(a-b)$.

Dividing throughout by $(a-b)$, we get

$\frac{(a-b)^2}{(a-b)} - \frac{2(a-b)}{(a-b)}$

$= a - b - 2$

$\therefore (a-b)^2 - 2(a-b) = (a-b)(a-b-2)$

Answer 1G.

$2a(p^2 + q^2) + 4b(p^2 + q^2)$

Here, the common factor is $2(p^2 + q^2)$.

Dividing throughout by $2(p^2 + q^2)$, we get

$\frac{2a(p^2 + q^2)}{2(p^2 + q^2)} + \frac{4b(p^2 + q^2)}{2(p^2 + q^2)}$

$= a + 2b$

$\therefore 2a(p^2 + q^2) + 4b(p^2 + q^2) = 2(p^2 + q^2)(a + 2b)$

Answer 1H.

$81(p+q)^2 - 9p - 9q$

$= 81(p+q)^2 - 9(p+q)$

Here, the common factor is $9(p+q)$.

Dividing throughout by $9(p+q)$, we get

$$\frac{81(p+q)^2}{9(p+q)} - \frac{9(p+q)}{9(p+q)}$$

$= 9(p+q) - 1$

$\therefore 81(p+q)^2 - 9p - 9q = 9(p+q)[9(p+q) - 1]$

Answer 1I.

$(mx+ny)^2 + (nx-my)^2$

$= m^2x^2 + n^2y^2 + 2mnxy + n^2x^2 + m^2y^2 - 2mnxy$

$= m^2x^2 + n^2y^2 + n^2x^2 + m^2y^2$

$= m^2x^2 + n^2x^2 + m^2y^2 + n^2y^2$

$= x^2(m^2+n^2) + y^2(m^2+n^2)$

Here, the common factor is (m^2+n^2).

Dividing throughout by (m^2+n^2), we get

$$\frac{x^2(m^2+n^2)}{(m^2+n^2)} + \frac{y^2(m^2+n^2)}{(m^2+n^2)}$$

$= x^2 + y^2$

$\therefore (mx+ny)^2 + (nx-my)^2 = (m^2+n^2)(x^2+y^2)$

Answer 1J.

$36(x+y)^3 - 54(x+y)^2$

Here, the common factor is $18(x+y)^2$.

Dividing throughout by $18(x+y)^2$, we get

$$\frac{36(x+y)^3}{18(x+y)^2} - \frac{54(x+y)^2}{18(x+y)^2}$$

$= 2(x+y) - 3$

$\therefore 36(x+y)^3 - 54(x+y)^2 = 18(x+y)^2[2(x+y) - 3]$

Answer 1K.

$p(p^2 + q^2 - r^2) + q(r^2 - q^2 - p^2) - r(p^2 + q^2 - r^2)$

$= p(p^2 + q^2 - r^2) - q(-r^2 + q^2 + p^2) - r(p^2 + q^2 - r^2)$

$= p(p^2 + q^2 - r^2) - q(p^2 + q^2 - r^2) - r(p^2 + q^2 - r^2)$

Here, the common factor is $(p^2 + q^2 - r^2)$.

Dividing throughout by $(p^2 + q^2 - r^2)$, we get

$$\frac{p(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)} - \frac{q(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)} - \frac{r(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)}$$

$= p - q - r$

$\therefore p(p^2 + q^2 - r^2) + q(r^2 - q^2 - p^2) - r(p^2 + q^2 - r^2) = (p^2 + q^2 - r^2)(p - q - r)$

Answer 2A.

$15xy - 9x - 25y + 15$

$= (15xy - 9x) - (25y + 15)$

$= 3x(5y - 3) - 5(5y - 3)$

$= (5y - 3)(3x - 5)$

Answer 2B.

$15x^2 + 7y - 3x - 35xy$

$= 15x^2 - 3x - 35xy + 7y$

$= (15x^2 - 3x) - (35xy - 7y)$

$= 3x(5x - 1) - 7y(5x - 1)$

$= (5x - 1)(3x - 7y)$

Answer 2C.

$9 + 3xy + x^2y + 3x$

$= 9 + 3xy + 3x + x^2y$

$= (9 + 3xy) + (3x + x^2y)$

$= 3(3 + xy) + y(3 + xy)$

$= (3 + xy)(3 + x)$

Answer 2D.

$8(2a + b)^2 - 8a - 4b$

$= 8(2a + b)^2 - (8a + 4b)$

$= 8(2a + b)^2 - 4(2a + b)$

$= 4(2a + b)[2(2a + b) - 1]$

$= 4(2a + b)[4a + 2b - 1]$

Answer 2E.

$x(x - 4) - x + 4$

$= x(x - 4) - 1(x - 4)$

$= (x - 4)(x - 1)$

Answer 2F.

$2m^3 - 5n^2 - 5m^2n + 2mn$

$= 2m^3 + 2mn - 5m^2n - 5n^2$

$= (2m^3 + 2mn) - (5m^2n + 5n^2)$

$= 2m(m^2 + n) - 5n(m^2 + n)$

$= (m^2 + n)(2m - 5n)$

Answer 2H.

$9x^3 + 6x^2y^2 - 4y^3 - 6xy$

$= 9x^3 + 6x^2y^2 - 6xy - 4y^3$

$= (9x^3 + 6x^2y^2) - (6xy + 4y^3)$

$= 3x^2(3x + 2y^2) - 2y(3x + 2y^2)$

$= (3x + 2y^2)(3x^2 - 2y)$

Answer 2I.

$3ax^2 - 5bx^2 + 9az^2 + 6ay^2 - 10by^2 - 15bz^2$

$= 3ax^2 + 6ay^2 + 9az^2 - 5bx^2 - 10by^2 - 15bz^2$

$= (3ax^2 + 6ay^2 + 9az^2) - (5bx^2 + 10by^2 + 15bz^2)$

$= 3a(x^2 + 2y^2 + 3z^2) - 5b(x^2 + 2y^2 + 3z^2)$

$= (x^2 + 2y^2 + 3z^2)(3a - 5b)$

Answer 2J.

$8x^3 - 24x^2y + 54xy^2 - 162y^3$

$= (8x^3 - 24x^2y) + (54xy^2 - 162y^3)$

$= 8x^2(x - 3y) + 54y^2(x - 3y)$

$= (x - 3y)(8x^2 + 54y^2)$

Answer 2K.

$2a + b + 3c - d + (2a + b)^3 + (2a + b)^2(3c - d)$

$= (2a + b + 3c - d) + [(2a + b)^3 + (2a + b)^2(3c - d)]$

$= 1(2a + b + 3c - d) + (2a + b)^2(2a + b + 3c - d)$

$= (2a + b + 3c - d)[1 + (2a + b)^2]$

Answer 2L.

$xy(a^2 + 1) + a(x^2 + y^2)$

$= a^2xy + xy + ax^2 + ay^2$

$= (a^2xy + ax^2) + (ay^2 + xy)$

$= ax(ay + x) + y(ay + x)$

$= (ay + x)(ax + y)$

Answer 2M.

$p^2x^2 + (px^2 + 1)x + p$

$= p^2x^2 + px^3 + x + p$

$= (p^2x^2 + px^3) + (p + x)$

$= px^2(p + x) + 1(p + x)$

$= (p + x)(px^2 + 1)$

Answer 2N.

$x^2 - (p + q)x + pq$

$= x^2 - px - qx + pq$

$= (x^2 - px) - (qx + pq)$

$= x(x - p) - q(x - p)$

$= (x - p)(x - q)$

Answer 2O.

$$p^2+\frac{1}{p^2}-2-5p+\frac{5}{p}$$
$$=\left(p^2+\frac{1}{p^2}-2\right)-\left(5p-\frac{5}{p}\right)$$
$$=\left((p)^2+\left(\frac{1}{p}\right)^2-2\times p\times\frac{1}{p}\right)-\left(5p-\frac{5}{p}\right)$$
$$=\left(p-\frac{1}{p}\right)^2-5\left(p-\frac{1}{p}\right)$$
$$=\left(p-\frac{1}{p}\right)\left(p-\frac{1}{p}-5\right)$$

Answer 2P.

$x+y+m(x+y)$
$=(x+y)+m(x+y)$
$=(x+y)(1+m)$

Answer 2Q.

$$\frac{1}{25x^2}+16x^2+\frac{8}{5}-12x-\frac{3}{5x}$$
$$=\left(\frac{1}{25x^2}+16x^2+\frac{8}{5}\right)-\left(12x+\frac{3}{5x}\right)$$
$$=\left(\left(\frac{1}{5x}\right)^2+(4x)^2+2\times\frac{1}{5x}\times 4x\right)-\left(12x+\frac{3}{5x}\right)$$
$$=\left(\frac{1}{5x}+4x\right)^2-3\left(4x+\frac{1}{5x}\right)$$
$$=\left(\frac{1}{5x}+4x\right)^2-3\left(\frac{1}{5x}+4x\right)$$
$$=\left(\frac{1}{5x}+4x\right)\left(\frac{1}{5x}+4x-3\right)$$

Answer 2R.

$2p(a^2-2b^2)-14p+(a^2-2b^2)^2-7(a^2-2b^2)$
$=2p(a^2-2b^2)+(a^2-2b^2)^2-14p-7(a^2-2b^2)$
$=[2p(a^2-2b^2)+(a^2-2b^2)^2]-[14p+7(a^2-2b^2)]$
$=(a^2-2b^2)(2p+a^2-2b^2)-7(2p+a^2-2b^2)$
$=(2p+a^2-2b^2)(a^2-2b^2-7)$

Ex 5.2

Answer 1Q.

$x^2 + 6x + 8$

$= x^2 + 4x + 2x + 8$

$= x(x + 4) + 2(x + 4)$

$= (x + 4)(x + 2)$

Answer 1B.

$x^2 - 11x + 24$

$= x^2 - 8x - 3x + 24$

$= x(x - 8) - 3(x - 8)$

$= (x - 8)(x - 3)$

Answer 1C.

$x^2 + 5x - 6$

$= x^2 + 6x - x - 6$

$= x(x + 6) - 1(x + 6)$

$= (x + 6)(x - 1)$

Answer 1D.

$p^2 - 12p - 64$

$= p^2 - 16p + 4p - 64$

$= p(p - 16) + 4(p - 16)$

$= (p - 16)(p + 4)$

Answer 1E.

$y^2 - 2y - 24$

$= y^2 - 6y + 4y - 24$

$= y(y - 6) + 4(y - 6)$

$=(y - 6)(y + 4)$

Answer 1F.

$3x^2 + 19x - 14$

$= 3x^2 + 21x - 2x - 14$

$= 3x(x + 7) - 2(x + 7)$

$= (x + 7)(3x - 2)$

Answer 1G.

$15a^2 - 14a - 16$

$= 15a^2 - 24a + 10a - 16$

$= 3a(5a - 8) + 2(5a - 8)$

$= (5a - 8)(3a + 2)$

Answer 1H.

$12 + x - 6x^2$

$= 12 + 9x - 8x - 6x^2$

$= 3(4 + 3x) - 2x(4 + 3x)$

$= (4 + 3x)(3 - 2x)$

Answer 1I.

$7x^2 + 40x - 12$

$= 7x^2 + 42x - 2x - 12$

$= 7x(x + 6) - 2(x + 6)$

$= (x + 6)(7x - 2)$

Answer 2A.

$5x^2 - 17xy + 6y^2$

$= 5x^2 - 15xy - 2xy + 6y^2$

$= 5x(x - 3y) - 2y(x - 3y)$

$= (x - 3y)(5x - 2y)$

Answer 2B.

$9x^2 - 22xy + 8y^2$

$= 9x^2 - 18xy - 4xy + 8y^2$

$= 9x(x - 2y) - 4y(x - 2y)$

$= (x - 2y)(9x - 4y)$

Answer 2C.

$2x^3 + 5x^2y - 12xy^2$

$= 2x^3 + 8x^2y - 3x^2y - 12xy^2$

$= 2x^2(x + 4y) - 3xy(x + 4y)$

$= (x + 4y)(2x^2 - 3xy)$

$= (x + 4y)x(2x - 3y)$

$= x(x + 4y)(2x - 3y)$

Answer 2D.

$x^2y^2 + 15xy - 16$

$= x^2y^2 + 16xy - xy - 16$

$= xy(xy + 16) - 1(xy + 16)$

$= (xy + 16)(xy - 1)$

Answer 2E.

$(2p + q)^2 - 10p - 5q - 6$

$= (2p + q)^2 - (10p - 5q) - 6$

$= (2p + q)^2 - 5(2p + q) - 6$

$= (2p + q)^2 - 6(2p + q) + (2p + q) - 6$

$= (2p + q)(2p + q - 6) + 1(2p + q - 6)$

$= (2p + q - 6)(2p + q + 1)$

Answer 2F.

$y^2 + 3y + 2 + by + 2b$

$= y^2 + y + 2y + 2 + by + 2b$

$= y^2 + y + by + 2y + 2 + 2b$

$= y(y + 1 + b) + 2(y + 1 + b)$

$= (y + 1 + b)(y + 2)$

Answer 2G.

$x^3y^3 - 8x^2y^2 + 15xy$

$= x^3y^3 - 3x^2y^2 - 5x^2y^2 + 15xy$

$= x^2y^2(xy - 3) - 5xy(xy - 3)$

$= (xy - 3)(x^2y^2 - 5xy)$

$= (xy - 3)xy(xy - 5)$

$= xy(xy - 3)(xy - 5)$

Answer 2H.

$6\sqrt{3}x^2 - 19x + 5\sqrt{3}$

$= 6\sqrt{3}x^2 - 10x - 9x + 5\sqrt{3}$

$= 2x\left(3\sqrt{3}x - 5\right) - \sqrt{3}\left(3\sqrt{3}x - 5\right)$

$= \left(3\sqrt{3}x - 5\right)\left(2x - \sqrt{3}\right)$

Answer 2I.

$2\sqrt{5}x^2 - 7x - 3\sqrt{5}$

$= 2\sqrt{5}x^2 - 10x + 3x - 3\sqrt{5}$

$= 2\sqrt{5}x\left(x - \sqrt{5}\right) + 3\left(x - \sqrt{5}\right)$

$= \left(x - \sqrt{5}\right)\left(2\sqrt{5}x + 3\right)$

Answer 3A.

$5(3x + y)^2 + 6(3x + y) - 8$

$= 5(3x + y)^2 + 10(3x + y) - 4(3x + y) - 8$

$= 5(3x + y)(3x + y + 2) - 4(3x + y + 2)$

$= (3x + y + 2)[5(3x + y) - 4]$

Answer 3B.

$5 - 4(a - b) - 12(a - b)^2$

$= 5 - 10(a - b) + 6(a - b) - 12(a - b)^2$

$= 5[1 - 2(a - b)] + 6(a - b)[1 - 2(a - b)]$

$= [5 + 6(a - b)][1 - 2(a - b)]$

$= (5 + 6a - 6b)(1 - 2a + 2b)$

Answer 3C.

$(3a - 2b)^2 + 3(3a - 2b) - 10$

$= (3a - 2b)^2 + 5(3a - 2b) - 2(3a - 2b) - 10$

$= (3a - 2b)(3a - 2b + 5) - 2(3a - 2b + 5)$

$= (3a - 2b + 5)((3a - 2b - 2)$

Answer 3D.

$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

$= (a^2 - 2a)^2 - 15(a^2 - 2a) - 8(a^2 - 2a) + 120$

$= (a^2 - 2a)(a^2 - 2a - 15) - 8(a^2 - 2a - 15)$

$= (a^2 - 2a - 15)(a^2 - 2a - 8)$

$= (a^2 - 5a + 3a - 15)(a^2 - 4a + 2a - 8)$

$= [a(a - 5) + 3(a - 5)][a(a - 4) + 2(a - 4)]$

$= [(a - 5)(a + 3)][(a - 4)(a + 2)]$

$= (a - 5)(a + 3)(a - 4)(a + 2)$

$= (a + 2)(a + 3)(a - 4)(a - 5)$

Answer 3E.

$(x + 4)^2 - 5xy - 20y - 6y^2$

$= (x + 4)^2 - 5y(x + 4) - 6y^2$

$= (x + 4)^2 - 6y(x + 4) + y(x + 4) - 6y^2$

$= (x + 4)(x + 4 - 6y) + y(x + 4 - 6y)$

$= (x + 4 - 6y)(x + 4 + y)$

$= (x - 6y + 4)(x + y + 4)$

Answer 3F.

$7(x - 2)^2 - 13(x - 2) - 2$

$= 7(x - 2)^2 - 14(x - 2) + (x - 2) - 2$

$= 7(x - 2)(x - 2 - 2) + 1(x - 2 - 2)$

$= 7(x - 2)(x - 4) + 1(x - 4)$

$= (x - 4)[7(x - 2) + 1]$

$= (x - 4)(7x - 14 + 1)$

$= (x - 4)(7x - 13)$

Answer 3G.

$12 - (y + y^2)(8 - y - y^2)$

$= 12 - a(8 - a)$ [Taking $y + y^2 = a$]

$= 12 - 8a + a^2$

$= 12 - 6a - 2a + a^2$

$= 6(2 - a) - a(2 - a)$

$= (2 - a)(6 - a)$

$= [2 - (y + y^2)][6 - (y + y^2)]$

$= (2 - y - y^2)(6 - y - y^2)$

$= (2 - 2y + y - y^2)(6 - 3y + 2y - y^2)$

$= [2(1 - y) + y(1 - y)][3(2 - y) + y(2 - y)]$

$= [(1 - y)(2 + y)][(2 - y)(3 + y)]$

$= (1 - y)(2 + y)(2 - y)(3 + y)$

$= (y - 1)(y + 2)(y - 2)(y + 3)$

Answer 3H.

$(p^2 + p)^2 - 8(p^2 + p) + 12$

$= (p^2 + p)^2 - 6(p^2 + p) - 2(p^2 + p) + 12$

$= (p^2 + p)(p^2 + p - 6) - 2(p^2 + p - 6)$

$= (p^2 + p - 6)(p^2 + p - 2)$

$= (p^2 + 3p - 2p - 6)(p^2 + 2p - p - 2)$

$= [p(p + 3) - 2(p + 3)][p(p + 2) - 1(p + 2)]$

$= [(p + 3)(p - 2)][(p + 2)(p - 1)]$

$= (p + 3)(p - 2)(p + 2)(p - 1)$

Answer 4A.

$(y^2 - 3y)(y^2 - 3y + 7) + 10$

$= a(a + 7) + 10$ [taking $(y^2 - 3y) = a$]

$= a^2 + 7a + 10$

$= a^2 + 5a + 2a + 10$

$= a(a + 5) + 2(a + 5)$

$= (a + 5)(a + 2)$

$= (y^2 - 3y + 5)(y^2 - 3y + 2)$

$= (y^2 - 3y + 5)(y^2 - 2y - y + 2)$

$= (y^2 - 3y + 5)[y(y - 2) - 1(y - 2)]$

$= (y^2 - 3y + 5)[(y - 2)(y - 1)]$

$= (y - 1)(y - 2)(y^2 - 3y + 5)$

Answer 4B.

$(t^2 - t)(4t^2 - 4t - 5) - 6$

$= (t^2 - t)[4(t^2 - t) - 5] - 6$

$= a[4a - 5] - 6$ [Taking $(t^2 - t) = a$]

$= 4a^2 - 5a - 6$

$= 4a^2 - 8a + 3a - 6$

$= 4a(a - 2) + 3(a - 2)$

$= (a - 2)(4a + 3)$

$= (t^2 - t - 2)[4(t^2 - t) + 3]$

$= (t^2 - 2t + t - 2)(4t^2 - 4t + 3)$

$= [t(t - 2) + 1(t - 2)](4t^2 - 4t + 3)$

$= [(t - 2)(t + 1)](4t^2 - 4t + 3)$

$= (t + 1)(t - 2)(4t^2 - 4t + 3)$

Answer 4C.

$12(2x - 3y)^2 - 1(2x - 3y) - 1$

$= 12a^2 - a - 1$ [Taking $(2x - 3y) = a$]

$= 12a^2 - 4a + 3a - 1$

$= 4a(3a - 1) + 1(3a - 1)$

$= (3a - 1)(4a + 1)$

$= [3(2x - 3y) - 1][4(2x - 3y) + 1]$

$= (6x - 9y - 1)(8x - 12y + 1)$

Answer 4D.

$6 - 5x + 5y + (x - y)^2$

$= 6 - 5(x - y) + (x - y)^2$

$= 6 - 3(x - y) - 2(x - y) + (x - y)^2$

$= 3[2 - (x - y)] - (x - y)[2 - (x - y)]$

$= 3(2 - x + y) - (x - y)(2 - x + y)$

$= (2 - x + y)(3 - x + y)$

Answer 4E.

$$2x^2 + \frac{x}{6} - 1$$

$$= \frac{1}{6}\left(12x^2 + x - 6\right)$$

$$= \frac{1}{6}\left(12x^2 + 9x - 8x - 6\right)$$

$$= \frac{1}{6}\left[3x(4x + 3) - 2(4x + 3)\right]$$

$$= \frac{1}{6}\left[(4x + 3)(3x - 2)\right]$$

$$= \frac{1}{6}(4x + 3)(3x - 2)$$

Answer 4F.

$P^4 + 23p^2q^2 + 90q^4$

$= p^4 + 18p^2q^2 + 5p^2q^2 + 90q^4$

$= p^2(p^2 + 18q^2) + 5q^2(p^2 + 18q^2)$

$= (p^2 + 18q^2)(p^2 + 5q^2)$

Answer 4G.

$2a^3 + 5a^2b - 12ab^2$

$= 2a^3 + 8a^2b - 3a^2b - 12ab^2$

$= 2a^2(a + 4b) - 3ab(a + 4b)$

$= (a + 4b)(2a^2 - 3ab)$

$= (a + 4b)a(2a - 3b)$

$= a(a + 4b)(2a - 3b)$

Ex 5.3

Answer 1A.

$x^2 - 16$

$= x^2 - 4^2$

$= (x - 4)(x + 4)$

Answer 1B.

$64x^2 - 121y^2$

$= (8x)^2 - (11y)^2$

$= (8x - 11y)(8x + 11y)$

Answer 1C.

$441 - 81y^2$

$= (21)^2 - (9y)^2$

$= (21 - 9y)(21 + 9y)$

$= 3(7 - 3y)3(7 + 3y)$

$= 9(7 - 3y)(7 + 3y)$

Answer 1D.

$x^6 - 196$

$= (x^3)^2 - (14)^2$

$= (x^3 - 14)(x^3 + 14)$

Answer 1E.

$625 - b^2$

$= (25)^2 - (b)^2$

$= (25 - b)(25 + b)$

Answer 1F.

$$m^2 - \frac{1}{9}n^2$$

$$= m^2 - \left(\frac{1}{3}n\right)^2$$

$$= \left(m - \frac{1}{3}n\right)\left(m + \frac{1}{3}n\right)$$

Answer 1G.

$8xy^2 - 18x^3$

$= 2x(4y^2 - 9x^2)$

$= 2x[(2y)^2 - (3x)^2]$

$= 2x[(2y - 3x)(2y + 3x)]$

$= 2x(2y - 3x)(2y + 3x)$

Answer 1H.

$16a^4 - 81b^4$

$= (4a^2)^2 - (9b^2)^2$

$= (4a^2 - 9b^2)(4a^2 + 9b^2)$

$= [(2a)^2 - (3b)^2](4a^2 + 9b^2)$

$= [(2a - 3b)(2a + 3b)](4a^2 + 9b^2)$

$= (2a - 3b)(2a + 3b)(4a^2 + 9b^2)$

Answer 1I.

$a(a - 1) - b(b - 1)$

$= a^2 - a - b^2 + b$

$= a^2 - b^2 - a + b$

$= (a^2 - b^2) - (a - b)$

$= (a - b)(a + b) - (a - b)$

$= (a - b)(a + b - 1)$

Answer 1J.

$(x + y)^2 - 1$

$= (x + y)^2 - (1)^2$

$= (x + y + 1)(x + y - 1)$

Answer 1K.

$x^2 + y^2 - z^2 - 2xy$

$= x^2 + y^2 - 2xy - z^2$

$= (x^2 + y^2 - 2xy) - z^2$

$= (x - y)^2 - (z)^2$

$= (x - y - z)(x - y + z)$

Answer 1L.

$(x - 2y)^2 - z^2$

$= (x - 2y)^2 - (z)^2$

$= (x - 2y - z)(x - 2y + z)$

Answer 2A.

$9(a - b)^2 - (a + b)^2$

$= [3(a - b)]^2 - (a + b)^2$

$= [3(a - b) - (a + b)][3(a - b) + (a + b)]$

$= (3a - 3b - a - b)(3a - 3b + a + b)$

$= (2a - 4b)(4a - 2b)$

$= 2(a - 2b)2(2a - b)$

$= 4(a - 2b)(2a - b)$

Answer 2B.

$25(x - y)^2 - 49(c - d)^2$

$= [5(x - y)]^2 - [7(c - d)]^2$

$= [5(x - y) - 7(c - d)][5(x - y) + 7(c - d)]$

$= (5x - 5y - 7c + 7d)(5x - 5y + 7c - 7d)$

Answer 2C.

$(2a - b)^2 - 9(3c - d)^2$

$= (2a - b)^2 - [3(3c - d)]^2$

$= [(2a - b) - 3(3c - d)][(2a - b) + 3(3c - d)]$

$= (2a - b - 9c + 3d)(2a - b + 9c - 3d)$

Answer 2D.

$b^2 - 2bc + c^2 - a^2$

$= (b^2 - 2bc + c^2) - a^2$

$= (b - c)^2 - (a)^2$

$= (b - c - a)(b - c + a)$

Answer 2E.

$$x^2 + \frac{1}{x^2} - 2$$

$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}$$

$$= \left(x - \frac{1}{x}\right)^2$$

$$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Answer 2F.

$(x^2 + y^2 - z^2)^2 - 4x^2y^2$

$= (x^2 + y^2 - z^2)^2 - (2xy)^2$

$= (x^2 + y^2 - z^2 - 2xy)(x^2 + y^2 - z^2 + 2xy)$

$= [(x^2 + y^2 - 2xy) - z^2][(x^2 + y^2 + 2xy) - z^2]$

$= [(x - y)^2 - z^2][(x + y)^2 - z^2]$

$= [(x - y - z)(x - y + z)][(x + y - z)(x + y + z)]$

$= (x - y - z)(x - y + z)(x + y - z)(x + y + z)$

Answer 2G.

$a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$

$= (a^2 + b^2 + 2ab) - (c^2 + d^2 + 2cd)$

$= (a + b)^2 - (c + d)^2$

$= (a + b + c + d)(a + b - c - d)$

Answer 2H.

$4xy - x^2 - 4y^2 + z^2$

$= z^2 - x^2 - 4y^2 + 4xy$

$= z^2 - (x^2 + 4y^2 - 4xy)$

$= z^2 - (x - 2y)^2$

$= [z - (x - 2y)][z + (x - 2y)]$

$= (z - x + 2y)(z + x - 2y)$

Answer 2I.

$4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$

$= (4x^2 - 12ax + 9a^2) - (y^2 + z^2 + 2yz)$

$= (2x - 3a)^2 - (y + z)^2$

$= [(2x - 3a) + (y + z)][(2x - 3a) - (y + z)]$

$= (2x - 3a + y + z)(2x - 3a - y - z)$

Answer 2J.

$(x + y)^3 - x - y$

$= (x + y)(x + y)^2 - (x + y)$

$= (x + y)[(x + y)^2 - 1]$

$= (x + y)[(x + y + 1)(x + y - 1)]$

$= (x + y)(x + y + 1)(x + y - 1)$

Answer 2K.

$y^4 + y^2 + 1$

$= y^4 + 2y^2 + 1 - y^2$

$= (y^2 + 1)^2 - y^2$

$= (y^2 + 1 + y)(y^2 + 1 - y)$

Answer 2L.

$(a^2 - b^2)(c^2 - d^2) - 4abcd$

$= a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2 - 4abcd$

$= a^2c^2 + b^2d^2 - 2abcd - a^2d^2 - b^2c^2 - 2abcd$

$= (a^2c^2 + b^2d^2 - 2abcd) - (a^2d^2 + b^2c^2 + 2abcd)$

$= (ac - bd)^2 - (ad + bc)^2$

$= [(ac - bd) + (ad + bc)][(ac - bd) - (ad + bc)]$

$= (ac - bd + ad + bc)(ac - bd - ad - bc)$

Answer 3A.

$(x^2 - 2x + 3)(x^2 + 2x + 3)$

$= [x^2 + 3 - 2x][x^2 + 3 + 2x]$

$= [(x^2 + 3) - 2x][(x^2 + 3) + 2x]$

$= (x^2 + 3)^2 - (2x)^2$

$= (x^2 + 3)^2 - 4x^2$

Answer 3B.

$$(x^2-2x+3)(x^2-2x-3)$$
$$=\left[(x^2-2x)+3\right]\left[(x^2-2x)+3\right]$$
$$=(x^2-2x)^2-(3)^2$$
$$=(x^2-2x)^2-9$$

Answer 3C.

$$(x^2+2x-3)(x^2-2x+3)$$
$$=\left[x^2+(2x-3)\right]\left[x^2-(2x-3)\right]$$
$$=x^2-(2x-3)^2$$
$$=x^4-(2x-3)^2$$

Answer 4A.

$$y^2+\frac{1}{4y^2}+1-6y-\frac{3}{y}$$
$$=\left(y^2+\frac{1}{4y^2}+1\right)-\left(6y+\frac{3}{y}\right)$$
$$=\left(y+\frac{1}{2y}\right)^2-6\left(y+\frac{1}{2y}\right)$$
$$=\left(y+\frac{1}{2y}\right)\left(y+\frac{1}{2y}-6\right)$$

Answer 4B.

$$4a^2+\frac{1}{4a^2}-2-6a+\frac{3}{2a}$$
$$=\left(4a^2+\frac{1}{4a^2}-2\right)-\left(6a-\frac{3}{2a}\right)$$
$$=\left(2a-\frac{1}{2a}\right)^2-3\left(2a-\frac{1}{2a}\right)$$
$$=\left(2a-\frac{1}{2a}\right)\left(2a-\frac{1}{2a}-3\right)$$

Answer 4C.

$$x^4 + y^4 - 6x^2y^2$$
$$= (x^2)^2 + (y^2)^2 - 2x^2y^2 - 4x^2y^2$$
$$= \left[(x^2)^2 + (y^2)^2 - 2x^2y^2\right] - (4x^2y^2)$$
$$= (x^2 - y^2)^2 - (2xy)^2$$
$$= (x^2 - y^2 - 2xy)(x^2 - y^2 + 2xy)$$

Answer 4D.

$$4x^4 + 25y^4 + 19x^2y^2$$
$$= 4x^4 + 25y^4 + 20x^2y^2 - x^2y^2$$
$$= (2x^2)^2 + (5y^2)^2 + 2 \times (2x^2) \times (5y^2) - x^2y^2$$
$$= \left[(2x^2)^2 + (5y^2)^2 + 2 \times (2x^2) \times (5y^2)\right] - x^2y^2$$
$$= \left[2x^2 + 5y^2\right] - (xy)^2$$
$$= (2x^2 + 5y^2 - xy)(2x^2 - 5y + xy)$$

Answer 4E.

$$p^2 + \frac{1}{p^2} - 3$$
$$= p^2 + \frac{1}{p^2} - 2 - 1$$
$$= \left(p^2 + \frac{1}{p^2} - 2 \times p \times \frac{1}{p}\right) - 1$$
$$= \left(p - \frac{1}{p}\right)^2 - 1^2$$
$$= \left(p - \frac{1}{p} + 1\right)\left(p - \frac{1}{p} - 1\right)$$

Answer 4F.

$$5x^2 - y^2 - 4xy + 3x - 3y$$
$$= x^2 + 4x^2 - y^2 - 4xy + 3x - 3y$$
$$= (x^2 - y^2) + (4x^2 - 4xy) + (3x - 3y)$$
$$= (x - y)(x + y) + 4x(x - y) + 3(x - y)$$
$$= (x - y)[(x + y) + 4x + 3]$$
$$= (x - y)(x + y + 4x + 3)$$
$$= (x - y)(5x + y + 3)$$